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Single Pass Fit Program (SPFP)

OCTOBER 1967

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AEROSPACE CORPORATION

Prepared for SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

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FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. F04695-67-C-0158.

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

for Robert G Lindsay, Col.

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Colonel, USAF
Director for Development
AF Satellite Control Facility

ABSTRACT

The Single Pass Fit Program (SPFP) is designed for use on the CDC 6400 or 6600 machine as the principal computational tool for preliminary orbit determination and processing of single arcs of raw tracking data. By an iterative differential correction procedure, it solves for a set of trajectory parameters, namely, initial position and velocity (and drag coefficient if desired) of the vehicle, which minimize the differences between measured and computed observation measurements. A comprehensive description of the mathematical model implemented in the SPFP is presented with emphasis placed on key characteristics, such as reference systems and equations, data processing and self-initializing techniques, residual editing, and differential correction. In addition, a basic usage guide is given with complete instructions for input data preparation and program operation.

CONTENTS

CONTENTS (Continued)

CONTENTS (Continued)

FIGURES

TABLES

NOMENCLATURE

A	azimuth observation measurement
[A]	matrix of partial derivatives
$\underline{B}$	vector of weighted residuals
$C_D A/W$	drag coefficient
E	elevation observation measurement
ECI	earth-centered inertial
$\underline{F}$	perturbative acceleration vector
G	matrix of constraints
O	set of refined observations
R	range observation measurement
$\dot{R}$	range rate observation measurement
$\underline{U}$	unit observation vector
V	magnitude of the vehicle velocity vector
$\underline{W}$	station frame vector
[W]	weighting matrix
X, Y, Z	principal cartesian (ECI) axes
a	semimajor axis
b	semiminor axis
e	eccentricity
f, g	analytic two-body scalar functions
h	height
i	inclination

NOMENCLATURE (Cont'd)

N	total number of observations
n	number of accepted observations
o	observation measurement
$\underline{r}$	vehicle position vector
r	magnitude of the vehicle position vector
$\dot{\underline{r}}$	vehicle velocity vector
t	time
v	true anomaly
x, y, z	cartesian (ECI) coordinates
α	right ascension
β	flight path angle
$\underline{\gamma}$	vector of parameters
δ	declination
ϵ	small positive quantity
λ	geodetic longitude
μ	gravitational constant
ρ	atmospheric density
σ	weighting factor
τ	time of perigee passage
ϕ'	geodetic latitude
ω	argument of perigee, or rotation rate
Ω	right ascension of the ascending node

I. INTRODUCTION

A. PURPOSE, SCOPE, AND LIMITATIONS

This document is intended to serve as a technical reference manual and basic usage guide for the Single Pass Fit Program (SPFP). Preliminary orbit determination, as defined in Section II, is a process for estimating from tracking data the initial conditions (position and velocity of a vehicle) for the differential equations of motion. The determination process implemented in the SPFP utilizes a precision numerical integration method to generate the vehicle's trajectory and associated partial derivatives, and an estimation technique that determines the initial conditions by a bounded least-squares differential correction procedure. A comprehensive description of the mathematical model used is presented with emphasis placed on key program characteristics.

Sections III (Reference Systems and Equations), IV (Functional Components), and V (Program Usage) are sufficiently detailed to meet the needs of Aerospace Corporation technical staff members concerned with preliminary orbit determination. In the interest of technical completeness, the user is referred to the TRACE Orbit Determination Program, Version D (Ref. 1), which covers those theoretical concepts omitted in this document.

B. HISTORICAL BACKGROUND

In the past, orbit determination has been performed by the generalized TRACE-D Orbit Determination Program at Aerospace Corporation. The SPFP was developed because of the increasing need for a specialized, compact, and rapid computational tool to accommodate preliminary orbit determination and processing of single arcs of raw tracking data. Its design characteristics (written entirely in 6600 FORTRAN) closely parallel those of TRACE-66, the latest version of TRACE.

C. SUMMARY OF KEY FUNCTIONS

By an iterative differential correction procedure, the SPFP solves for a set of parameters, namely, position and velocity (and drag coefficient if desired) of the vehicle that minimize the differences between measured and computed tracking observations from single arcs of data. The preliminary orbit determination process used in SPFP can be identified by the following functions:

1. Data processing refines the raw observation measurements by conversion and pre-editing (or rejection) criteria. Range and elevation measurements are subject to maximum and minimum constraints, and scaling and refraction corrections.
2. Self-initializing of the initial position and velocity from selected data points is performed with a gaussian iterative scheme.
3. Trajectory generation of position and velocity, and associated partial derivatives at each data point, is accomplished by numerical integration of the equations of motion in a Cowell formulation with an eighth-order differencing method.
4. Residual evaluation (that is, measurement residuals) is made from the differences between observed and computed values.
5. Observation partial derivative evaluation is the computation of the derivative of each data measurement with respect to trajectory parameters.
6. Residual editing is performed on those points whose residuals exceed a predetermined multiple of the prior iterative rms (root mean square) value.
7. Matrix accumulation consists of storing observation partials and residuals in a matrix A, which is used to form the weighted normalized symmetric matrix product A^TA.
8. Differential correction of parameters depends upon a bounded least-squares process that minimizes the sum-of-squares of the observation residuals.

SECTION II

PRELIMINARY ORBIT DETERMINATION

A. THE BASIC PROBLEM AND KEY CHARACTERISTICS OF ORBIT DETERMINATION

Fundamentally, orbit determination is a process of statistically estimating from tracking data the initial state vector of a vehicle, that is of determining the initial conditions (position and velocity vectors) for the vector differential equation of motion. The basic problem of preliminary orbit determination is to establish a process for formulating an approximation of the initial state vector. Many such processes exist at various levels of complexity. In the case of the Single Pass Fit Program, the preliminary determination of satellite orbits from single arcs of data is accomplished with a weighted least-squares differential correction procedure.

Orbit determination is characterized by the trajectory and partial derivative generation methods, and by the mathematical scheme that estimates the initial state vector of the vehicle from differences between the observed and computed measurements. For preliminary orbit determination, the SPFP utilizes a precision numerical integration method to generate the trajectory and associated partial derivatives; estimation of the initial state vector is performed by a bounded least-squares differential correction procedure that uses residual editing techniques to enhance convergence.

B. THE PRELIMINARY DETERMINATION MODEL

The computational model used in SPFP to determine orbits from tracking data can be characterized by four specific elements: simulated vehicle environment, parameter specification, data and residual editing, and weights.

1. SIMULATED VEHICLE ENVIRONMENT

To simulate the environment in which a vehicle is moving, the following physical forces are considered to be acting:

- Gravitational attraction of an aspherical earth derived from a generalized potential function.
- Atmospheric drag (U.S. Standard Atmosphere 1962, COESA, density model, Ref. 2).

Inclusion of any subset of these forces is completely at the discretion of the program user through input options.

2. PARAMETER SPECIFICATION

The SPFP is specifically designed to determine by differential correction the initial state vector S_o $(\underline{r}, \dot{\underline{r}})$ and, if desired, the drag coefficient $C_D A/W$.

3. DATA AND RESIDUAL EDITING

The following are raw earth-fixed data observations accepted by SPFP:

- Slant range (R)
- Local azimuth (A)
- Local elevation (E)
- Range rate ($\dot{R}$)

These are subject to preprocessing editing criteria, such as maximum-minimum range and elevation, and data arc (pass) duration. Also, each time a differential correction cycle is performed, the residuals are subject to editing criteria determined by the root-mean-square (rms) value for all residuals in the previous iteration.

4. WEIGHTS

In general, measurements are assigned a priori weights according to tracking-station and observation-measurement type. The twofold purpose of the weights is to normalize units so that different types of measurements can be mixed in the least squares process, and to adjust their relative influence in fitting measurements of different accuracies.

SECTION III

REFERENCE SYSTEMS AND EQUATIONS

The basic reference systems and equations used in the computational method of the SPFP are now defined and discussed, preceded by the symbols used throughout this report.

A. SYMBOLS

$\cdot$	time differentiation and vector dot production operation
$\cdot\cdot$	time differentiation
$\_$	a vector; for example, $\underline{r}$ is the vehicle position vector
( )	relates to a functional relationship, such as $S(\underline{r}, \dot{\underline{r}})$ the vehicle state vector, which is a function of $\underline{r}$ and $\dot{\underline{r}}$, or $S(a, e, i, \Omega, \omega, \tau)$ the state vector expressed in terms of elliptical elements. Also, used to denote row vectors; for example, $\underline{r} = (x, y, z)$
~	raw observation or measurement
Δ	increment or difference
x	vector cross product operation
| |	absolute value, or vector magnitude operation; for example, $r = \|\underline{r}\|$
∇	vector gradient operator
{ }	a matrix, or a set of elements; for example, $\tilde{O}_i = \{ T_i, \tilde{R}_i, \tilde{A}_i, \tilde{E}_i, \tilde{\dot{R}}_i \}$ the set of raw measurements of the i^{th} observation
$\|\| \ \|\|$	the vector norm operator
I[ ]	the integer operator of a scalar

[ ] a matrix; for example,

$\underline{r} = [x, y, z]$ (row vector) or $\underline{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (column vector)

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3 by 3 identify matrix

$A = [A]$ 4 by 7 matrix of partials and residuals

Superscripts

*	particular value of a predetermined constant
s	the station
i	particular index relative to a set or vector
c	computed or simulated value
T	matrix transpose symbol

Subscripts

o	initial value or reference point
g	the Greenwich meridian
s	the station
i, j, k, ℓ	particular index denoting members of a set or components of a vector. In general, i, j, k, ℓ = 1, 2, . . .
e	the earth
α	a transformation that is a function of α
Φ	a transformation that is a function of Φ
γ	differentiation of a vector with respect to the parameter γ

B. REFERENCE SYSTEMS

Preliminary orbit determination is a computational process of coordinate transformations and requires selection of the proper reference systems for the performance of the necessary function. Several such systems are used in the SPFP; basic to all is the earth-centered inertial (ECI) system, which has as its fundamental plane and principal axis the true equator at epoch and the mean equinox at midnight of the date of epoch. This reference system does not account for precession and nutation effects.

1. EARTH-CENTERED INERTIAL SYSTEM

Inertial position, velocity, and acceleration vectors are oriented in the reference system illustrated in Figure 1.

The position and velocity components of a vehicle's state vector within this reference system may in addition be expressed in terms of the spherical or classical elliptic systems.

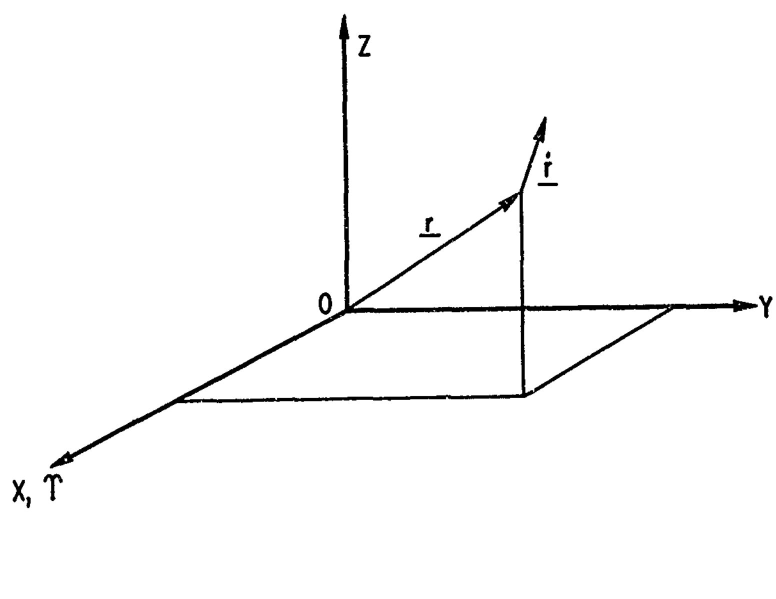

Figure 1. Earth-centered Inertial Reference System

In Figure 1:

O	is the origin, which coincides with the center of the earth
$\underline{r}$	is the radius vector (x, y, z)
$\underline{\dot{r}}$	is the velocity vector $(\dot{x}, \dot{y}, \dot{z})$
X	is the direction of the vernal equinox, ♈ in the equator plane
Y	is the right-hand axis to X and Z
Z	is the direction north perpendicular to the equator plane
x	is the position component in the X direction
y	is the position component in the Y direction
z	is the position component in the Z direction
$\dot{x}$	is the velocity component in the X direction
$\dot{y}$	is the velocity component in the Y direction
$\dot{z}$	is the velocity component in the Z direction

2. SPHERICAL SYSTEM

In the spherical system shown in Figure 2, the vehicle state vector at time t may be written as

$$S = S(\alpha, \delta, \beta, A, r, V)$$

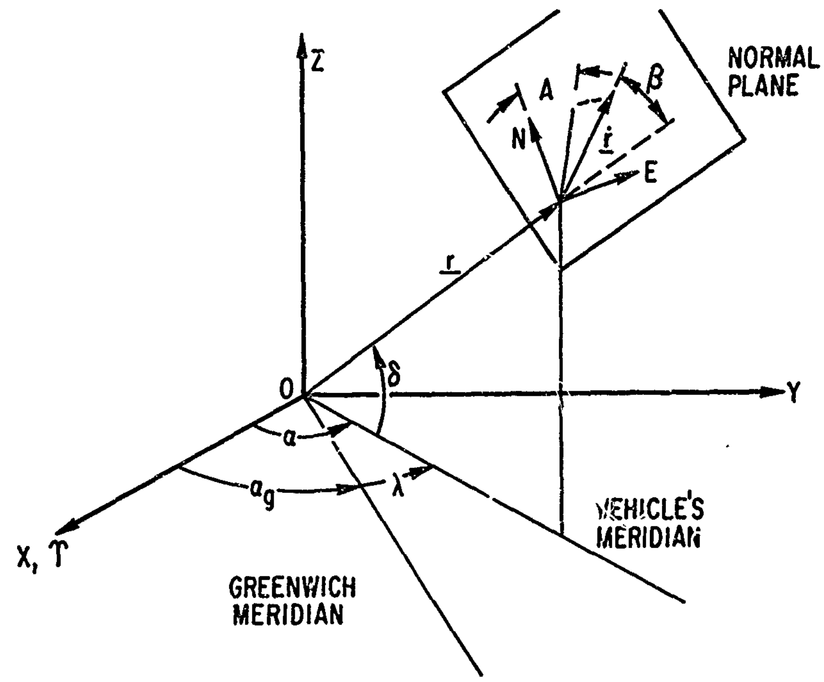

Figure 2. Spherical Coordinate Reference System

In Figure 2:

α	is the right ascension
δ	is the geocentric latitude
β	is the angle between radius and velocity vectors
A	is the azimuth angle between the projection of the velocity vector in the plane normal to the geocentric radius vector and the direction to due north
r	is the magnitude of the geocentric radius vector $\underline{r}$
V	is the magnitude of the velocity vector $\underline{\dot{r}}$
α_g	is the right ascension of Greenwich
λ	is the earth-fixed longitude of the vehicle

3. CLASSICAL ELLIPTIC SYSTEM

The state vector in the classical elliptic system (Fig. 3) is given at time t by

$$S = S(a, e, i, \Omega, \omega, \tau)$$

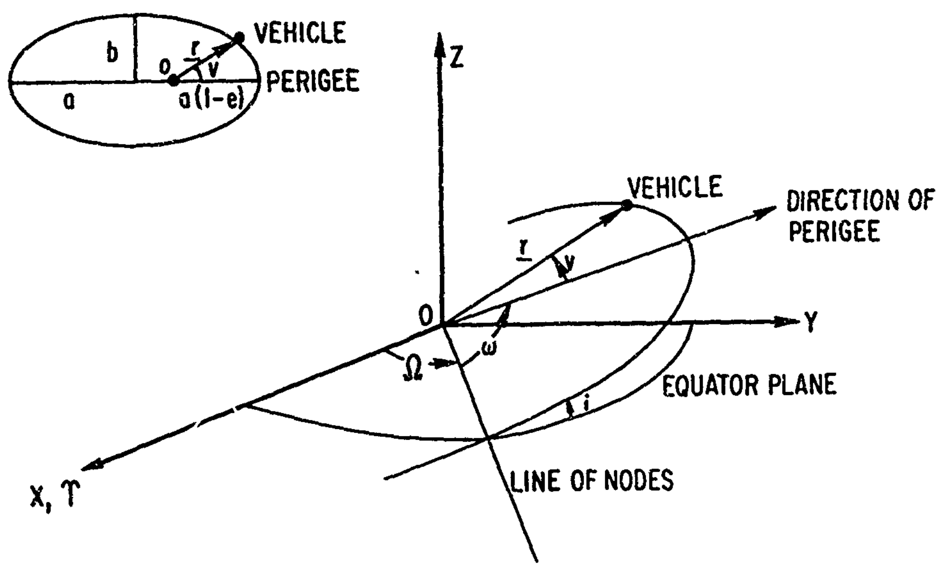

Figure 3. Classical Elliptic Reference Frame

In Figure 3:

a	is the semimajor axis of the osculating conic
e	is the eccentricity
i	is the inclination
Ω	is the right ascension of the ascending node
ω	is the angle between the direction of perigee and the line of nodes

τ	is the time of last perigee passage referenced to midnight of epoch date

v	is the true anomaly

b	is the semiminor axis

4. STATION SYSTEM (W-FRAME)

The station-dependent measurements are transformed to this system via an intermediate reference frame (Fig. 4) defined by the station coordinates (λ_s, ϕ', h). If W^s denotes the station, then the W-frame is the plane containing the vector $\underline{W}^s = (W_1^s, W_2^s, W_3^s)$, so that $W_2^s \equiv 0$ and W_3^s coincides with the Z axis.

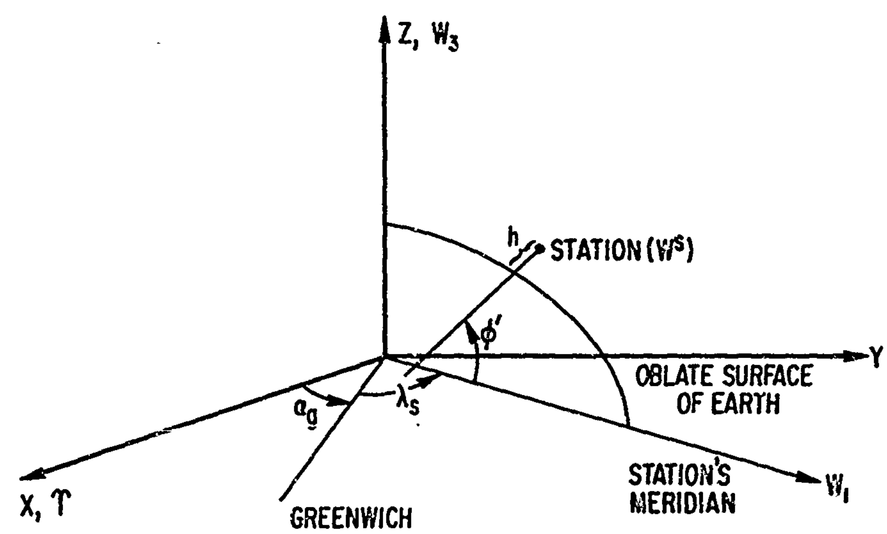

Figure 4. Station Reference Frame

In Figure 4:

λ_s	is the longitude of the station

ϕ'	is the geodetic latitude of the station

h	is the height of the station above the earth ellipsoid

5. OBSERVATION FRAME

The measurements of the vehicle's position with respect to the station are made in the frame defined by the local horizontal plane that is tangent to the earth ellipsoid (fig. 5).

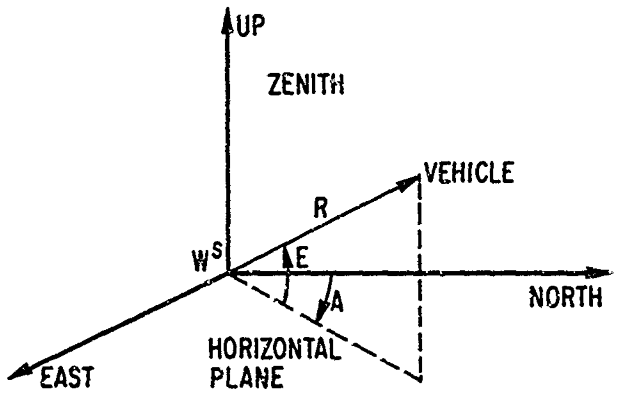

Figure 5. The Observation Frame

In Figure 5:

R is the distance between the station and the vehicle (range)

A is the angle between the projection of the range in the local horizon plane and due north

E is the angle between the projection and the range direction

W^S is the origin of the observation frame and is related to the station frame by the $\underline{W}^S$ vector

C. TRANSFORMATION EQUATIONS

The transformation of the state vector to the other systems involves the following equations.

1. FROM ECI TO THE SPHERICAL SYSTEM

$S(x, y, z, \dot{x}, \dot{y}, \dot{z})$ to $S(\alpha, \delta, \beta, A, r, V)$

$$\alpha = \arc \tan (y/x)$$

$$\delta = \arc \sin (z/r)$$

$$\beta = \arc \cos [(x\dot{x} + y\dot{y} + z\dot{z})/rV]$$

$$A = \arc \tan \left[r(x\dot{y} - y\dot{x})/[y(y\dot{z} - z\dot{y}) - x(z\dot{x} - x\dot{z})]\right]$$

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$V = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} \qquad (1)$$

2. FROM ECI TO THE CLASSICAL ELLIPTIC SYSTEM

$S(x, y, z, \dot{x}, \dot{y}, \dot{z})$ to $S(a, e, i, \Omega, \omega, \tau)$

$$a = (2/r - V^2/\mu)^{-1}$$

$$e = [(e \cos E)^2 + (e \sin E)^2]^{\frac{1}{2}}$$

$$i = \arc \tan \left[\left(P_1^2 + Q_1^2\right)^{\frac{1}{2}}/(P_1Q_2 - P_2Q_1)\right]$$

$$\Omega = \arc \tan [(P_2Q_3 - P_3Q_2)/(P_1Q_3 - P_3Q_1)]$$

$$\omega = \arc \tan (P_3/Q_3)$$

$$\tau = t - M/n \qquad (2)$$

where

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$V^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$$

$$e \cos E = 1 - \frac{r}{a}$$

$$e \sin E = (x\dot{x} + y\dot{y} + z\dot{z})/(|a|\mu)^{\frac{1}{2}}$$

$$p = [r^2V^2 - (x\dot{x} + y\dot{y} + z\dot{z})^2]/\mu$$

$$D = (x\dot{x} + y\dot{y} + z\dot{z})/e\mu$$

$$\dot{D} = e \cos E/er$$

$$H = (r - p)/e\sqrt{\mu p}$$

$$\dot{H} = (x\dot{x} + y\dot{y} + z\dot{z})/re\sqrt{\mu p}$$

$$P_1 = \dot{D}x - D\dot{x}$$

$$P_2 = \dot{D}y - D\dot{y}$$

$$P_3 = \dot{D}z - D\dot{z}$$

$$Q_1 = \dot{H}x - H\dot{x}$$

$$Q_2 = \dot{H}y - H\dot{y}$$

$$Q_3 = \dot{H}z - H\dot{z}$$

$$n = (\mu/|a|^3)^{\frac{1}{2}}$$

$$M = E - e \sin E$$

$$E = \arctan(e \sin E/e \cos E)$$

3. THE STATION COORDINATES TO THE W-FRAME

$$(\lambda_s, \phi', h) \qquad \text{to} \qquad (W_1^s, W_2^s, W_2^s)$$

$$W_1^s = (a_s a_e + h)\cos\phi'$$

$$W_2^s = 0$$

$$W_3^s = [(1 - f)^2 b_s a_e + h]\sin\phi' \tag{3}$$

where

a_e is the mean equatorial radius of the earth

f is the earth flattening coefficient

$a_s^{-2} = \cos^2\phi' + (1 - f)^2 \sin^2\phi'$

$b_s^{-2} = \sin^2\phi' + (1 - f)^2 \cos^2\phi'$

4. OBSERVATION MEASUREMENTS TO THE ECI SYSTEM

$$(R, A, E) \qquad \text{to} \qquad (x, y, z)$$

The transformation of observation measurements to the ECI system permits the computation of the initial state vector by a gaussian two-point scheme with an analytic two-body formulation. Thus

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 0 & -\sin\phi' & \cos\phi' \\ 1 & 0 & 0 \\ 0 & \cos\phi' & \sin\phi' \end{pmatrix} \begin{pmatrix} R\cos E \sin A \\ R\cos E\cos A \\ R\sin E \end{pmatrix} + \begin{pmatrix} W_1^s \\ W_2^s \\ W_3^s \end{pmatrix} \right] \tag{4}$$

where

$$\alpha = \alpha_{g_o} + \lambda_s + \omega_e(t - t_o)$$

where

α_{g_o} = the right ascension of Greenwich at midnight of epoch date

ω_e = the earth rotation rate

$t-t_o$ = the elapsed time since midnight of epoch date

5. THE STATE VECTOR TO THE W-FRAME

(x, y, z) to (W_1, W_2, W_3)

To compute simulated measurements, the state vector is transformed to the W-frame by

$$\underline{W} = \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{5}$$

$(\dot{x}, \dot{y}, \dot{z})$ to $(\dot{W}_1, \dot{W}_2, \dot{W}_3)$

$$\dot{\underline{W}} = \begin{pmatrix} \dot{W}_1 \\ \dot{W}_2 \\ \dot{W}_3 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} + \omega_e y \\ \dot{y} - \omega_e x \\ \dot{z} \end{pmatrix} \tag{6}$$

D. TRAJECTORY EQUATIONS

Three sets of trajectory equations are used in the SPFP to compute the vehicle state vector, $S(\underline{r}, \dot{\underline{r}})$, or associated partial derivatives of the state at some time t. These equation sets are described by their functional usage in Table I.

Table I. Functional Use of Equation Sets

Equation Set	Usage Function
Two-body Equations	To evaluate the state vector $S_o(\underline{r}, \dot{\underline{r}})$ at some specified epoch time t_o from initial conditions given at observation time t_k
Equations of Motion	To generate the state vector $S(\underline{r}, \dot{\underline{r}})$ at each observation time t_i
Variational Equations	To generate partial derivatives of the state vector with respect to the parameter vector $\underline{y}$ at each observation time t_i

1. TWO-BODY EQUATIONS

Exact closed-form expressions for the f and g series (Ref. 3) are utilized to evaluate the initial state vector $S_o(\underline{r}, \dot{\underline{r}})$ at some specified time t_o, if the position and velocity vectors at some time t_k are given. Let $\underline{r}_k$ and $\dot{\underline{r}}_k$ denote the given vector at t_k; then the initial state vector at t_o can be expressed as

$$\underline{r}_o = f\underline{r}_k + g\dot{\underline{r}}_k \tag{7}$$

and

$$\dot{\underline{r}}_o = \dot{f}\underline{r}_k + \dot{g}\dot{\underline{r}}_k \tag{8}$$

where f, g, $\dot{f}$, and $\dot{g}$ are scalar coefficients given by

$$f = \left(\frac{r_o}{r_k}\right)\cos\Phi - \left(\frac{\dot{r}_k}{h}\right) r_o \sin\Phi$$

$$g = \left(\frac{r}{h}\right)_k r_o \sin\Phi$$

$$\Phi = (v_o - v_k) = \text{the time anomaly difference}$$

$$(v_o - v_k) = \text{function of } [(E_o - E_k)] = \text{the eccentric anomaly difference}$$

$$n(t_o - t_k) = (E_o - E_k) + 2\left(\frac{r_k \dot{r}_k}{\sqrt{\mu a}}\right)\sin^2\left(\frac{E_o - E_k}{2}\right) - \left(1 - \frac{r_k}{a}\right)\sin(E_o - E_k)$$

$$a = \left(\frac{2}{r_k} - \frac{\dot{\underline{r}}_k \cdot \dot{\underline{r}}_k}{\mu}\right)^{-1}$$

$$n = \left(\frac{\mu}{a^3}\right)^{1/2}$$

μ = product of the Newtonian gravitational constant and mass of the earth

$$\dot{f} = \left(\frac{r_o}{r_k}\right)(\dot{r}_o \cos\Phi - r_o \sin\Phi\,\dot{\Phi}) - \left(\frac{\dot{r}}{h}\right)_k (\dot{r}_o \sin\Phi + r_o \cos\Phi\,\dot{\Phi})$$

$$\dot{g} = \left(\frac{r}{h}\right)_k (\dot{r}_o \sin\Phi + r_o \cos\Phi\,\dot{\Phi})$$

$$\dot{\Phi} = \frac{h}{r_o^2}$$

$$h = |\underline{r}_k \times \dot{\underline{r}}_k|$$

2. EQUATIONS OF MOTION

The state vector $S(\underline{r}, \dot{\underline{r}})$ is generated at each observation time by numerically integrating the equations of motion in a Cowell formulation with time as the independent variable. The equations of motion are expressed by the second-order vector differential equation

$$\ddot{\underline{r}} = -\frac{\mu \underline{r}}{r^3} + \underline{F} \tag{9}$$

with the initial values

$$\underline{r}(t_o) = \underline{r}_o \quad , \quad \dot{\underline{r}}(t_o) = \dot{\underline{r}}_o$$

where

- $\underline{r}$ is a 3-vector of ECI components of position (x, y, z)
- r is the magnitude of $\underline{r}$ ($r = |\underline{r}|$)
- μ is the product (GM) of the Newtonian gravitational constant and mass of the earth
- $\underline{F}$ is the perturbative acceleration

In particular, the primary term $-\mu\underline{r}/r^3$ is the inverse square central force due to gravitation. The perturbative term $\underline{F}$ is the sum of the gravitational acceleration due to the noncentral force field of the earth $\underline{F}_A$ and the acceleration due to atmospheric drag $\underline{F}_B$.

$\underline{F}_A$ is derived from the generalized potential function

$$U = \frac{\mu}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a_e}{r}\right)^n \sum_{m=0}^{n} P_n^m (\sin \phi)(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right] \tag{10}$$

where

μ is the product of the Newtonian gravitational constant and the earth mass

r, ϕ, λ are the geocentric distance, latitude, and (east) longitude of the vehicle

a_e is the mean equatorial radius of the earth

P_n^m is the Legendre associated function of the first kind of degree n and order m

C_{nm}, S_{nm} are numerical coefficients

The vector form of $\underline{F}_A$ can be expressed as

$$\underline{F}_A = \nabla U(\underline{r}) + \frac{\mu \underline{r}}{r^3} \tag{11}$$

and $\underline{F}_B$, the force due to atmospheric drag, is expressed by the vector equation

$$\underline{F}_B = - \rho \left(\frac{V_A}{2}\right) \left(\frac{C_D A}{W}\right) \dot{\underline{r}}_A \tag{12}$$

where

ρ is the density at height h above an oblate earth

$C_D A/W$ is the drag coefficient

$\dot{\underline{r}}_A$ is the vehicle velocity vector relative to a rotating atmosphere ($\dot{\underline{r}}_A = [\dot{x}_A, \dot{y}_A, \dot{z}_A]$)

V_A $= |\dot{\underline{r}}_A|$

where

$$\dot{x}_A = \dot{x} + \omega_a y$$

$$\dot{y}_A = \dot{y} - \omega_a x$$

$$\dot{z}_A = \dot{z}$$

ω_a = rotation rate of the atmosphere

The density model used to obtain ρ in the SPFP is the 1962 U.S. Standard Atmosphere (Ref. 2), which is designed to represent atmospheric density under average conditions of solar activity through middle-latitude bands. It is similar to the ARDC 1959 model (Ref. 4) in which the sequence of connected linear segments involving variation of molecular scale temperature with altitude is considered. The 1962 model reflects density values somewhat lower than the ARDC 1959 at altitudes above 200 km.

3. VARIATIONAL EQUATIONS

To generate partial derivatives of the state vector with respect to a parameter set $\underline{y}$, variational equations are numerically integrated and then the partial derivatives are interpolated at each observation time. In the SPFP only parameters associated with the equations of motion are considered; in particular, initial condition parameters, such as $S_o(\underline{r}, \dot{\underline{r}})$ and $C_D A/W$, are the only ones specified in the differential correction process.

If the parameter set is denoted by the vector $\underline{y}$, it can be functionally related to the vector equation of motion by the form of Eq. (9) as

$$\ddot{\underline{r}} = \ddot{\underline{r}}(\underline{r}, \dot{\underline{r}}, \underline{y}, t)$$

Let $\underline{r}_\gamma$ denote $\partial\underline{r}/\partial\gamma$; then the general vector variational equation is

$$\ddot{\underline{r}}_\gamma = \left[\frac{\partial}{\partial\underline{r}}\left(-\frac{\mu\underline{r}}{r^3}\right) + \frac{\partial\underline{F}}{\partial\underline{r}}\right]\underline{r}_\gamma + \left[\frac{\partial\underline{F}}{\partial\dot{\underline{r}}}\right]\dot{\underline{r}}_\gamma + \frac{\partial\underline{F}}{\partial\gamma} \tag{13}$$

with the necessary initial conditions

$$\underline{r}_\gamma(t_o) \quad , \quad \dot{\underline{r}}_\gamma(t_o) \tag{14}$$

depending on the parameter γ of $\underline{y}$. The $\partial\underline{F}/\partial\gamma$ in Eq. (13) is called the non-homogeneous term of the variational equation for γ of $\underline{y}$. Note that the contents of the brackets in Eq. 13 are 3 by 3 matrices. The matrix in the first set of brackets is evaluated as the sum of $[V] + [T]$, where

$$[V] = \frac{\partial}{\partial\underline{r}}\left(-\frac{\mu\underline{r}}{r^3}\right) + \frac{\partial\underline{F}_A}{\partial\underline{r}} \tag{15}$$

and

$$[T] = \frac{\partial\underline{F}_B}{\partial\underline{r}} \tag{16}$$

represent the dependence of the gravitational and atmospheric drag accelerations upon the position of the vehicle. The matrix $[\partial\underline{F}/\partial\dot{\underline{r}}]$ can be written as

$$\left[\frac{\partial\underline{F}_B}{\partial\dot{\underline{r}}}\right] = -\frac{1}{2}\rho V_A\left(\frac{C_D A}{W}\right)\left[\frac{\dot{\underline{r}}_A\dot{\underline{r}}_A^{\,T}}{V_A^2} + I\right] \tag{17}$$

a. Initial Conditions for Variational Equations

Initial conditions for the second-order linear vector differential given in Eq. (14) are

$$\left[\frac{\partial \underline{r}}{\partial \underline{r}_o}\right]_{t_o} = \left[\frac{\partial \underline{\dot{r}}}{\partial \underline{\dot{r}}_o}\right]_{t_o} = I \qquad \text{(3 by 3 identify matrix)}$$

$$\left[\frac{\partial \underline{r}}{\partial \underline{\dot{r}}_o}\right]_{t_o} = \left[\frac{\partial \underline{\dot{r}}}{\partial \underline{r}_o}\right]_{t_o} = [0] \qquad \text{(zero matrix)}$$

where $\underline{\gamma} = (\underline{r}_o, \underline{\dot{r}}_o)$, representing $S_o(\underline{r}, \underline{\dot{r}})$. For γ of $\underline{\gamma}$ equal to $(C_D A/W)$

$$\left[\frac{\partial \underline{r}}{\partial C_D A/W}\right]_{t_o} = \left[\frac{\partial \underline{\dot{r}}}{\partial C_D A/W}\right]_{t_o} = [0]$$

b. Nonhomogeneous Term for Variational Equation

The nonhomogeneous term $\partial \underline{F}/\partial \gamma$ in Eq. (13) is equal to zero, if $\gamma = x_o, y_o, z_o, \dot{x}_o, \dot{y}_o$, or $\dot{z}_o$; and equals $\underline{F}_B(C_D A/W)^{-1}$ only if $\gamma = (C_D A/W)$, a non-zero quantity.

E. OBSERVATION MEASUREMENTS AND ASSOCIATED PARTIAL DERIVATIVES

Equations for the computation of simulated measurements and their associated partial derivatives are formulated directly from the $\underline{W}^s$ vector.

SIMULATED OR COMPUTED MEASUREMENTS

$$\text{Range (R)} = \left(Q_1^2 + Q_2^2 + Q_3^2\right)^{\frac{1}{2}}$$

$$\text{Azimuth (A)} = \text{arc tan}\,(v_1/v_2)$$

(cont)

$$\text{Elevation (E)} = \arc\tan\left[v_3/\left(v_1^2 + v_2^2\right)^{\frac{1}{2}}\right]$$

$$\text{Range Rate } (\dot{R}) = U_1\dot{W}_1 + U_2\dot{W}_2 + U_3\dot{W}_3 \qquad (18)$$

where

$$\underline{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix} = \underline{W} - \underline{W}^s = \begin{pmatrix} W_1 - W_1^s \\ W_2 \\ W_3 - W_3^s \end{pmatrix}$$

$$\underline{U} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \frac{1}{R} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -\sin\phi' & 0 & \cos\phi' \\ \cos\phi' & 0 & \sin\phi' \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}$$

ASSOCIATED PARTIAL DERIVATIVES OF THE MEASUREMENT WITH RESPECT TO THE PARAMETER VECTOR

Range $\dfrac{\partial R}{\partial \gamma} = \underline{U} \cdot \dfrac{\partial \underline{Q}}{\partial \gamma} = \underline{U} \cdot \dfrac{\partial \underline{W}}{\partial \gamma}$

Azimuth $\dfrac{\partial A}{\partial \gamma} = \dfrac{1}{R\cos E}\left[\dfrac{\partial W_2}{\partial \gamma}\cos A + \left(-\dfrac{\partial W_1}{\partial \gamma}\sin\phi' + \dfrac{\partial W_3}{\partial \gamma}\cos\phi'\right)\cos A\right]$

(cont)

$$\text{Elevation} \quad \frac{\partial E}{\partial \gamma} = \frac{1}{R \cos E}\left[\frac{\partial W_1}{\partial \gamma} \cos \phi' + \frac{\partial W_3}{\partial \gamma} \sin \phi' - \frac{\partial R}{\partial \gamma} \sin E\right]$$

$$\text{Range Rate} \; \frac{\partial \dot{R}}{\partial \gamma} = \frac{\partial \underline{W}}{\partial \gamma} \cdot \underline{\dot{U}} + \frac{\partial \underline{\dot{W}}}{\partial \gamma} \cdot \underline{U} \qquad (19)$$

where γ is a representative element of the parameter vector $\underline{\gamma}$.

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SECTION IV

FUNCTIONAL COMPONENTS

Seven basic functional components identify the computational model used in the SPFP to perform preliminary orbit determination. Figure 6 schematically illustrates the function of each component in relation to the solution of the preliminary orbit reconstruction problem. These components are defined by the key functions they perform, as shown in Table II.

Table II. Key Functions of Basic SPFP Components

COMPONENT	KEY FUNCTION
Input Data Processor	Process input data
Data Processor	Convert and refine raw observation measurements
Self-Initializor for the State Vector	Estimate the initial state vector from selected data points
Trajectory Generator	Produce the state vector and its associated partial derivatives at each data point
Residual and Partial Derivative Evaluator	Evaluate data residuals and data partial derivatives
Residual Editor and Matrix Accumulator	Edit data based on residual level, and accumulate matrix of normalized partial derivatives A^TA
Differential Corrector	Perform least square calculations to compute the parameter correction vector $\Delta\underline{\gamma}$

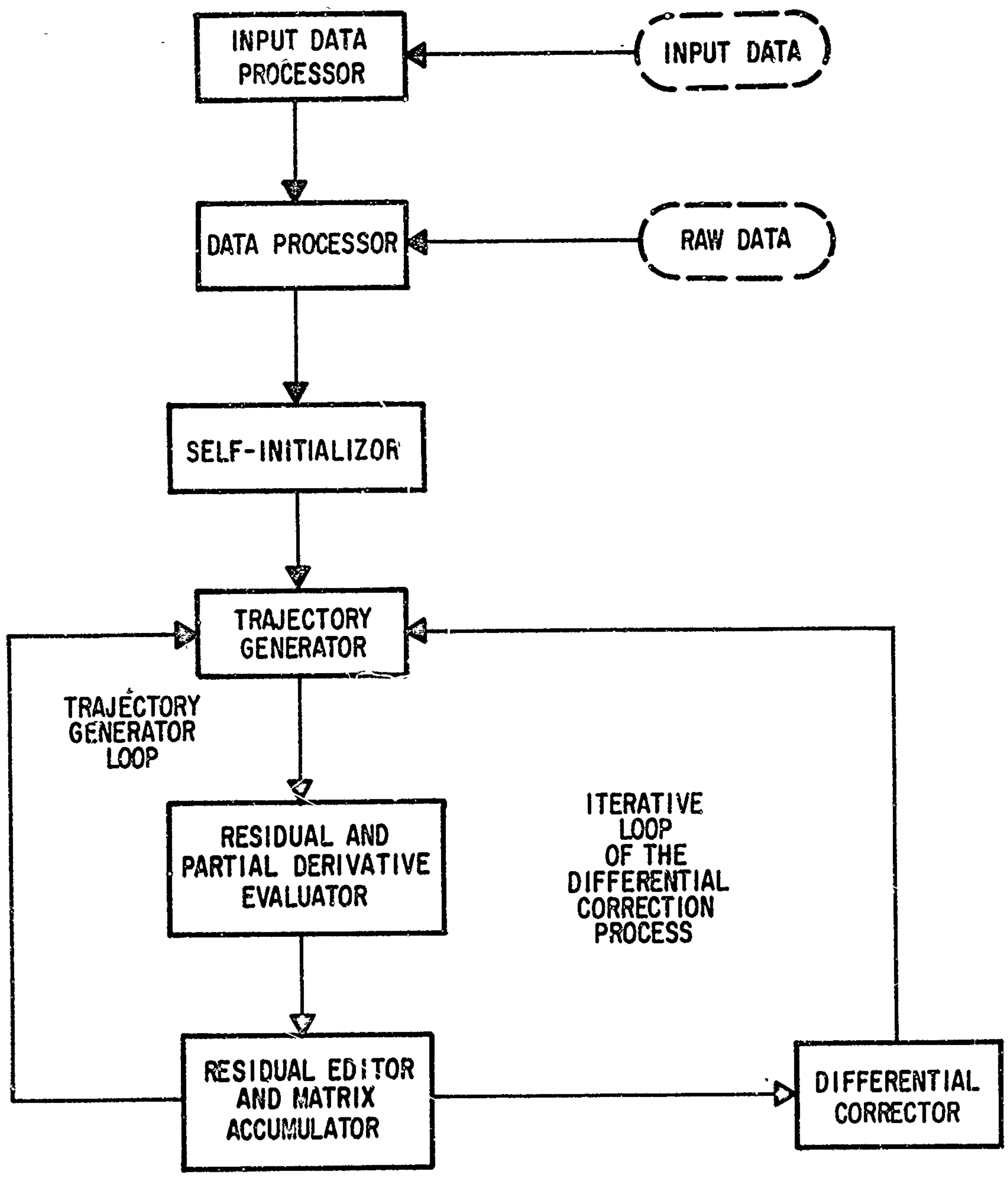

Figure 6. Schematic of the Prelimin ıry Orbit Determination Prc. ess

A. INPUT DATA PROCESSOR

The basic function of the input data processor is to initialize the observation-dependent quantities and model-dependent variables.

1. OBSERVATION-DEPENDENT QUANTITIES

The observation-dependent quantities of the SPFP consist of station location and observation information, editing criteria, iteration and initial condition selection controls, and conversion factors.

Station location and observation information is initialized for each single arc of tracking data. This consists of items such as:

- Earth-fixed station location coordinates
- Measurement reference data
- Observation data format and measurement codes.

Editing criteria include quantities such as

- Range and elevation refraction corrections
- Range scaling factor
- Maximum-minimum range and elevation constraints
- Initial residual limits for each measurement.

Iteration and initial condition selection controls consist of:

- Maximum number of iterations
- Data point selection index for self-determination of initial state vector
- Measurement weighting sigmas
- Bounds for corrections to the state vector estimate
- Scaling factors that reduce (increase) bounds on a diverging (converging) iteration
- Relative convergence criterion.

Conversion factors, such as distance in feet per earth radii (ft/er) and velocity (ft/sec/er/min) constants, are necessary for input-output.

2. MODEL-DEPENDENT VARIABLES

The model-dependent variables of the SPFP consist of physical constants, numerical integration controls, and force model specifications.

Physical constants — geopotential and atmospheric model constants — that define the physical environment of the vehicle are:

- Gravitational constant (μ)
- Numerical geopotential coefficients (C_{nm}, S_{nm})
- Mean equatorial radius (a_e)
- Earth flattening term (f)
- Atmospheric constants.

Numerical integration controls consist of:

- Initial integration step size
- Maximum and minimum allowable step size
- Ratio of Runge-Kutta-to-Cowell step size

The force modal is further specified by the user through data inputs such as:

- Number of terms in geopotential model
- Geopotential normalization
- Drag coefficient ($C_D A/W$)

B. DATA PROCESSOR

Raw observation measurements are subject to conversion and pre-editing (or rejection) criteria through the SPFP data processor. The procedure used refines the raw observation set $\tilde{O}$ into an observation set O, which is directly utilized in the reconstruction scheme.

1. RAW OBSERVATION SET, $\tilde{O}$

Let the raw observation set $\tilde{O}$ be denoted as

$$\tilde{O} = \left\{ \{T_i, \tilde{R}_i, \tilde{A}_i, \tilde{E}_i, \tilde{\dot{R}}_i\} \quad : \quad i = 1, \ldots, N \right\} \tag{20}$$

where

N is the total number of observations for a given arc of data

T_i is the date and time of day of the i^{th} observation; that is, $T_i = \{YEAR, MONTH, DAY, HOUR, MINUTE, SECOND\}_i$

$\tilde{R}_i$ is the observed range (ft)

$\tilde{A}_i$ is the observed azimuth (deg)

$\tilde{E}_i$ is the observed elevation (deg)

$\tilde{\dot{R}}_i$ is the observed range rate (ft/sec)

i is the subscript indicating the i^{th} observation.

2. CONVERSION

Time of the i^{th} raw observation T_i is the first quantity to be refined. A reference Julian date (J.D.) is computed from T_i or is prespecified in order to redefine each T_i into t_i with units of minutes from midnight of the reference Julian date.

Observed measurements have the conversion units shown in Table III.

Table III. Observed Measurement Conversion Units

MEASUREMENT	RAW UNIT	REFINED UNIT
Range	feet	earth radii
Azimuth	degrees	radians
Elevation	degrees	radians
Range Rate	feet/sec	earth radii/minute

3. PRE-EDITING

In the usual reconstruction situation, range and elevation measurements are required to lie within specified limits. Thus each $\tilde{R}_i$ and $\tilde{E}_i$ are subjected to the following predetermined constraints for $i = 1, \ldots, N$:

R_{MIN}	Minimum range constraint
R_{MAX}	Maximum range constraint
E_{MIN}	Minimum elevation constraint

That is, any raw range or elevation measurement not satisfying the above constraints deletes its corresponding observation from $\tilde{O}$; hence, N is reduced by one for each rejected observation.

The data arc is sorted and examined to determine if $\Delta t_i = t_i - t_1$ $(i = 1, \ldots, N)$ is greater than some prespecified arc duration value, Δt^*. If $\Delta t_i > \Delta t^*$, then the i^{th} observation is deleted and N is reduced by one.

4. REFINED OBSERVATION SET O

After the measurements for each observation are converted and pre-edited, the range data is subject to scaling and refraction corrections upon option; in addition, a refraction correction may be applied to the elevation measurement. Therefore, upon option

$$E_i = \tilde{E}_i + \Delta E \quad , \qquad \text{if } \tilde{E}_i \leq 0.1 \text{ radian}$$

$$E_i = \tilde{E}_i + \Delta E^* \quad , \qquad \text{if } \tilde{E}_i > 0.1 \text{ radian}$$

where ΔE and ΔE^* denote predetermined refraction corrections.

The resultant set of observations is defined as the refined observation set O; that is

$$O = \left\{ \{t_i, R_i, A_i, E_i, \dot{R}_i\} \quad : \quad i = 1, \ldots, n \right\} \tag{21}$$

where

n	is the total number of acceptable observations
t_i	is the time of the i^{th} data point in minutes from midnight of the reference Julian date
R_i	is the range (er)

A_i is the azimuth (rad)

E_i is the elevation (rad)

$\dot{R}_i$ is the range rate (er/min)

i is the subscript indicating the i^{th} observation

with the set $O_i = \{t_i, R_i, A_i, E_i, \dot{R}_i\}$, denoting the i^{th} data point.

C. SELF-INITIALIZATION OF THE STATE VECTOR

The technique used in the SPFP to estimate the initial state vector from selected data points is a gaussian iterative scheme that requires two geocentric position vectors with associated times. The initialization procedure implemented consists of the following four steps:

1. Selection of two data points from the refined data set O; that is, O_k and O_j
2. Computation of two geocentric position vectors from selected data points; that is, $\underline{r}_k(t_k)$ and $\underline{r}_j(t_j)$
3. Application of the gaussian iterative scheme to obtain corresponding inertial velocity vector $\dot{\underline{r}}_k(t_k)$
4. Utilization of closed form expressions for the f and g series to compute the state vector at t_o.

1. DATA POINT SELECTION

Given the refined data set $O = \{O_i : i = 1, \ldots, n\}$ and a non-zero integer value for data point selection index m, the selection rule for choosing k and j is given in Table IV.

Table IV. Data Point Selection Rule

$m = 1$	,	$k = 1$ and $j = n$ (first and last)
$= 2$	,	$k = 1$ and $j = I[n/2]$ (first and midpoint)
$m \geq 3$	,	$k = I[n/m]$ and $j = I[n \cdot (m-1)/m]$

2. POSITION VECTORS FROM SELECTED POINTS

Inertial position vectors are computed from selected data points O_k and O_j by applying two transformations. If

$$O_i = \{t_i, R_i, A_i, E_i\} \qquad \text{for } i = k \text{ and } j$$

$$S = \{\Phi, \lambda, h\}$$

and

$$\underline{W}^s = \left(W_1^s, W_2^s, W_3^s\right)$$

then a unit station-vehicle vector $\underline{V}^i$ is computed where

$$\underline{V}^i = \begin{bmatrix} V_1^i \\ V_2^i \\ V_3^i \end{bmatrix} = \begin{bmatrix} \cos E_i \sin A_i \\ \cos E_i \cos A_i \\ \sin E_i \end{bmatrix} \qquad \text{for } i = k \text{ and } j$$

The T_Φ^s transformation matrix is applied to $\underline{V}^i$ to obtain vector $\underline{U}^i$; that is

$$\underline{U}^i = T_\Phi^s \underline{V}^i = \begin{bmatrix} U_1^i \\ U_2^i \\ U_3^i \end{bmatrix} \qquad \text{for } i = k \text{ and } j$$

where

$$T_{\Phi}^{s} = \begin{bmatrix} 0 & -\sin\Phi & \cos\Phi \\ 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \end{bmatrix}$$

This $\underline{U}^i$ vector is multiplied by the slant range R_i to obtain vector $\underline{Q}^i$; that is

$$\underline{Q}^i = R_i \underline{U}^i = \begin{bmatrix} Q_1^i \\ Q_2^i \\ Q_3^i \end{bmatrix} \qquad \text{for } i = k \text{ and } j$$

Vector $\underline{W}^i$ is computed by augmenting $\underline{Q}^i$ with $(W_1^s, 0, W_3^s)$; that is

$$\underline{W}^i = \begin{bmatrix} W_1^i \\ W_2^i \\ W_3^i \end{bmatrix} = \begin{bmatrix} Q_1^i + W_1^s \\ Q_2^i \\ Q_3^i + W_3^s \end{bmatrix} \qquad \text{for } i = k \text{ and } j$$

Finally, the inertial position vectors are obtained by applying the T_{α}^{s} transformation matrix to $\underline{W}^i$; that is

$$\underline{r}_i = T_{\alpha}^{s} \underline{W}^i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \qquad \text{for } i = k \text{ and } j$$

where

$$T_{\alpha}^{s} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\alpha = \alpha_{g_o} + \lambda + t_i \omega_e$$

Hence the set of boundary conditions $\{\underline{r}_k, \underline{r}_j, \Delta t\}$ is determined where

$\underline{r}_k$ is the geocentric position vector at observation time t_k

$\underline{r}_j$ is the geocentric position vector at observation time t_j

Δt is the time difference between the j^{th} and k^{th} data points ($\Delta t = t_j - t_k$, $k < j$)

3. GAUSSIAN ITERATIVE SCHEME TO OBTAIN VELOCITY VECTOR

To determine a (two-body) orbit satisfying the $\{\underline{r}_k, \underline{r}_j, \Delta t\}$ bounding conditions, the following gaussian iterative scheme (Ref. 5) is used in the SPFP to compute the inertial velocity vector $\dot{\underline{r}}_k$ from f ang g closed form expressions. Compute

$$\underline{W} = \frac{\underline{r}_j \times \underline{r}_k}{|\underline{r}_j \times \underline{r}_k|} = \begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix}$$

(cont.)

$$\tau = \mu^{1/2}(t_j - t_k) = \mu^{1/2}\Delta t$$

$$r_j = (\underline{r}_j \cdot \underline{r}_j)^{1/2}$$

$$r_k = (\underline{r}_k \cdot \underline{r}_k)^{1/2}$$

$$\cos(v_j - v_k) = (\underline{r}_k \cdot \underline{r}_j)/(r_k r_j) \qquad , \qquad 0 \le (v_j - v_k) < \pi$$

$$\sin(v_j - v_k) = \frac{(x_k v_j - x_j v_k)}{|x_k y_j - x_j y_k|} \cdot [1 - \cos^2(v_j - v_k)]^{1/2} \tag{22}$$

where $(v_j - v_k)$ is the true anomaly difference between observation times t_k and t_j, which corresponds to the in-plane angle difference for Δt.

If $W_z \geq 0$, then motion is direct; and if $W_z < 0$, motion is retrograde. Obtain constants ℓ and m by

$$\ell = \frac{r_j + r_k}{4(r_j \cdot r_k)^{1/2}\cos\left(\frac{v_j - v_k}{2}\right)} - \frac{1}{2}$$

$$m = \tau^2\left[2(r_j \cdot r_k)^{1/2}\cos\left(\frac{v_j - v_k}{2}\right)\right]^{-3} \tag{23}$$

Note that $\cos[(X - Y)/2] = +[(1 + \cos(X - Y))/2]^{1/2}$.

Choose $y_{(o)} = 1$, and then compute

$$x_{(i)} = \frac{m}{y_{(i)}^2} - \ell \tag{24}$$

$$\cos\left(\frac{E_j - E_k}{2}\right)_{(i)} = 1 - 2x_{(i)} \tag{25}$$

$$\sin\left(\frac{E_j - E_k}{2}\right)_{(i)} = [4x_{(i)}(1 - x_{(i)})]^{1/2} \tag{26}$$

Equations (25) and (26) determine $(E_j - E_k)_{(i)}$ uniquely.

Compute x from the $(E_j - E_k)_{(i)}$ value from

$$x = \frac{(E_j - E_k)_{(i)} - \sin(E_j - E_k)_{(i)}}{\sin^3\left(\frac{E_j - E_k}{2}\right)_{(i)}} \tag{27}$$

to obtain a new value for $y_{(i+1)}$; that is

$$y_{(i+1)} = 1 + x(\ell + x_{(i)}) \tag{28}$$

The iterative cycle (Eqs. (24) through (28))is repeated until $|\Delta y| = |y_{(i+1)} - y_{(i)}| \leq$ some predetermined ϵ.

Computation continues with

$$a^{1/2} = \tau\left[2y(r_k \cdot r_j)^{1/2}\cos\left(\frac{v_j - v_k}{2}\right)\sin\left(\frac{E_j - E_k}{2}\right)\right] \tag{29}$$

$$f = 1 - \frac{a}{r_j}[1 - \cos(E_j - E_k)] \tag{30}$$

$$g = \tau - a^{3/2}[(E_j - E_k) - \sin(E_j - E_k)] \tag{31}$$

$$\dot{\underline{r}}_k = \mu^{1/2}(\underline{r}_j - f\underline{r}_k)/g \tag{32}$$

Hence $\underline{r}_k$ and $\dot{\underline{r}}_k$ are known and the state vector is determined at time t_k.

4. STATE VECTOR AT t_o

Closed form expressions for the f and g series are used to evaluate the state vector at $I(t_1) = t_o$. If the position and velocity at t_k are given, then the state vector at t_o can be expressed as

$$\underline{r}_o = f\underline{r}_k + g\dot{\underline{r}}_k \tag{33}$$

and

$$\dot{\underline{r}}_o = \dot{f}\underline{r}_k + \dot{g}\dot{\underline{r}}_k \tag{34}$$

where

$$f = \left(\frac{r_o}{r_k}\right)\cos\Phi - \left(\frac{\dot{r}}{h}\right)_k r_o \sin\Phi$$

$$g = \left(\frac{r}{h}\right)_k r_o \sin\Phi$$

$$\Phi = v_o - v_k$$

$$\dot{f} = \left(\frac{r_o}{r_k}\right)(\dot{r}_o\cos\Phi - r_o\sin\Phi\dot{\Phi}) - \left(\frac{\dot{r}}{h}\right)_k(r_o\sin\Phi + r_o\cos\Phi\dot{\Phi})$$

$$\dot{g} = \left(\frac{r}{h}\right)_k(\dot{r}_o\sin\Phi + r_o\cos\Phi\dot{\Phi})$$

$$\dot{\Phi} = \frac{h}{r_o^2}$$

$$h = |\underline{r}_k \times \dot{\underline{r}}_k|$$

D. TRAJECTORY GENERATOR

Trajectory generation in the SPFP is accomplished by numerically integrating the equations of motion and associated variational equations in a Cowell formulation with an eighth-order differencing method (Ref. 6). The position and velocity components of the state vector and its associated partial derivatives are interpolated at each observation time t_i with a fifth-degree polynomial technique.

1. EQUATIONS OF MOTION

The motion of the vehicle with respect to time is described by the second-order vector differential equation

$$\underline{\ddot{r}} = -\frac{\mu \underline{r}}{r^3} + \underline{F} \tag{35}$$

with initial values

$$\underline{r}(t_o) = \underline{r}_o \quad , \quad \underline{\dot{r}}(t_o) = \underline{\dot{r}}_o$$

and where

- $\underline{r}$ is a 3-vector of rectangular components of position (x, y, z) in an ECI reference system
- r is the magnitude of $\underline{r}$ ($r = |\underline{r}|$)
- μ is the product (GM) of the Newtonian gravitational constant and mass of the earth
- $\underline{F}$ is the acceleration vector resulting from perturbing forces; i.e., all forces other than the inverse square central force due to gravitation.

In particular, the perturbative acceleration as modeled in the SPFP is the sum of the gravitational acceleration due the noncentral force field of the earth $\underline{F}_A$ and the acceleration due to atmospheric drag $\underline{F}_B$.

2. PARTIAL DERIVATIVES

Parameters associated with $\underline{\ddot{r}}$ may be functionally related to the state vector by the following form of Eq. (35)

$$\underline{\ddot{r}} = \frac{-\mu \underline{r}}{r^3} + \underline{F}(\underline{r}, \underline{\dot{r}}, \underline{\gamma}, t)$$

or

$$\underline{\ddot{r}} = \underline{\ddot{r}}(\underline{r}, \underline{\dot{r}}, \underline{\gamma}, t)$$

where $\underline{\gamma}$ is a vector of parameters.

If $\underline{r}_\gamma$ denotes $\partial \underline{r}/\partial \gamma$, this leads to the general vector variational equation form

$$\underline{\ddot{r}}_\gamma = \left[\frac{\partial}{\partial \underline{r}} \left(- \frac{\mu \underline{r}}{r^3} \right) + \frac{\partial \underline{F}}{\partial \underline{r}} \right] \underline{r}_\gamma + \left[\frac{\partial \underline{F}}{\partial \underline{\dot{r}}} \right] \underline{\dot{r}}_\gamma + \frac{\partial \underline{F}}{\partial \gamma}$$

with the necessary initial conditions $\underline{r}_\gamma(t_o)$, $\underline{\dot{r}}_\gamma(t_o)$, depending on the parameter γ of $\underline{\gamma}$. Numerical integration of this second-order vector variational equation yields the partial derivatives $\underline{r}_\gamma$ and $\underline{\dot{r}}_\gamma$ for each γ of $\underline{\gamma}$ that is a function of time and subject to its respective differential equation.

E. RESIDUAL AND PARTIAL DERIVATIVE EVALUATOR

Once the trajectory point at observation time t_i is determined, then data residual and data partial derivatives can be evaluated with respect to trajectory parameters. To accomplish this the state vector at time t_i is transformed into a data reference frame by the transformations given in Section III. C. 4, which allows an observation set of data measurements to be

computed. Let O_i^c denote this set; that is

$$O_i^c = \left\{ t_i, R_i^c, A_i^c, E_i^c, \dot{R}_i^c \right\} \quad , \tag{36}$$

where the computed measurements for the i^{th} observation are

R_i^c is the computed slant range

A_i^c is the computed azimuth

E_i^c is the computed elevation

$\dot{R}_i^c$ is the computed range rate

1. RESIDUAL COMPUTATION

A residual set $\Delta_i = O_i - O_i^c$ is computed from the refined observed data point O_i and the computed data point O_i^c at time t_i; that is,

$$\Delta_i = \left\{ \Delta R_i, \Delta A_i, \Delta E_i, \Delta \dot{R}_i \right\} \tag{37}$$

where

$$\Delta R_i = R_i - R_i^c$$

$$\Delta A_i = A_i - A_i^c$$

$$\Delta E_i = E_i - E_i^c$$

$$\Delta \dot{R}_i = \dot{R}_i - \dot{R}_i^c$$

These residuals of Δ_i are an index of observation measurement errors, force model errors, and vehicle initial state vector errors from observation to observation during the data pass. Ultimately, the differential correction process will depend on Δ_i for $i = 1, \ldots, n$.

2. DATA PARTIAL DERIVATIVES

Partial derivatives of each data measurement (R, A, E, $\dot{R}$) with respect to trajectory parameters are denoted in the chain rule expression

$$\frac{\partial O_i^c}{\partial \underline{Y}} = \frac{\partial O_i^c}{\partial \underline{r}} \frac{\partial \underline{r}}{\partial \underline{Y}} + \frac{\partial O_i^c}{\partial \underline{\dot{r}}} \frac{\partial \underline{\dot{r}}}{\partial \underline{Y}} ,$$

and are necessary for the differential correction process at each time t_i ($i = 1, \ldots, n$). The sequence of computations leading to these data partial derivatives is

a. Evaluation of partial derivatives of computed observation set O_c^i with respect to the state vector at time t_i; that is, $\partial O_i^c / \partial \underline{r}$ and $\partial O_i^c / \partial \underline{\dot{r}}$ (refer to partial derivative equations in Section III. E).

b. Evaluation of $\partial O_i^c / \partial \underline{Y}$ accomplished by using trajectory partials, $\partial \underline{r} / \partial \underline{Y}$ and $\partial \dot{r} / \partial \underline{Y}$, and the observation partials, $\partial O_i^c / \partial \underline{r}$ and $\partial O_i^c / \partial \underline{\dot{r}}$, to evaluate the chain rule expression.

The exact equations used to evaluate these data partials $\partial O_i^c / \partial \underline{Y}$ are given in Section III. E.

F. RESIDUAL EDITOR AND MATRIX ACCUMULATOR

Before the accumulation of observation partials and residuals in matrix form, the residuals are subject to the following editing technique.

1. RESIDUAL EDITING

As the residuals Δ_i for each observation are evaluated, their squares are accumulated. At the termination of the accumulation, the root mean square (rms) is calculated for each measurement type and used subsequently in the next accumulation to remove (edit) the data points whose residuals have exceeded a predetermined multiple of the rms value. The multiple rms factor is reduced for each iteration to improve convergence; hence, only the best observations contribute to the matrix accumulation.

2. MATRIX ACCUMULATION

Observation partials $\partial O_i^c / \partial \gamma$ and residuals Δ_i at time t_i are stored for each data measurement in a row of the accumulation matrix A. A typical row of A would be of the form

$$\left[\frac{\partial o_j^c}{\partial \gamma_1} \quad , \quad \frac{\partial o_j^c}{\partial \gamma_2} \quad , \quad \cdots \quad , \quad \frac{\partial o_j^c}{\partial \gamma_k} \quad , \quad \Delta_j \right]_{\text{Row } j \ (1 \leq j \leq 4)}$$

where o_j^c is the computed j^{th} measurement for which

$j = 1$, range

$j = 2$, azimuth

$j = 3$, elevation

$j = 4$, range rate

and

γ_k is the k^{th} parameter with $k = 1, \ldots, 7$

Δ_j is the residual of the j^{th} measurement.

The weighted normalized matrix product denoted by $A^T A$ is accumulated from the A matrix at each data point and a diagonal matrix W, which is used to normalize the units and give relative weights to each data measurement. The j, k element of the $A^T A$ is expressed as

$$(A^T A)_{jk} = \sum_{\ell=1}^{4} A_{\ell j} W_{\ell \ell} A_{\ell k}$$

$$= \sum_{\ell=1}^{4} \frac{\partial o_\ell^c}{\partial \gamma_j} \frac{1}{\sigma_\ell^2} \frac{\partial o_\ell^c}{\partial \gamma_k} \qquad \begin{array}{l} j = 1, \ldots, 7 \\ k = 1, \ldots, 7 \end{array}$$

where o_ℓ is the ℓ^{th} data measurement, γ_j and γ_k are two independent representatives of parameter vector $\underline{\gamma}$, and σ_ℓ is the weighting factor of the ℓ^{th} data measurement.

Note that the elements of the last column (or row) of the $A^T A$ are of the form

$$(A^T A)_{j8} = \sum_{\ell=1}^{4} \frac{\partial o_\ell^c}{\partial \gamma_j} \frac{1}{\sigma_\ell^2} \Delta o_\ell$$

which is denoted by $A^T \underline{B}$ and contains the nonhomogenous values of the linear system solved in the differential correction process.

G. DIFFERENTIAL CORRECTOR

Differential corrections $\Delta\underline{\gamma}$ to the parameters $\underline{\gamma}$ are obtained from the solution of the linear system described in Ref. 1 as

$$[A^T A + Z G^T G] \Delta\underline{\gamma} = A^T \underline{B} \tag{38}$$

where

- $A^T A$ is the weighted normalized symmetric matrix of the form $A^T WA$, with W denoting the diagonal weighting matrix and A being the matrix of observation partials and residuals
- Z is an optimizing coefficient to be solved for
- $G^T G$ is a diagonal matrix product of the form $(G^T G)_{jj} = g_j^2$ $(j = 1, \ldots, 7)$ where G is a diagonal matrix which specifies the upper bounds g_j to the corrections $\Delta\gamma_j (j = 1, \ldots, 7)$
- $\Delta\underline{\gamma}$ is the vector of parameter corrections to be solved for
- $\underline{B}$ is a vector of weighted residuals

The solution of this linear system, namely $\Delta \underline{y}$, is subject to the constraint that the optimizing coefficient Z has a value such that the corrections $\Delta \underline{y} = \Delta \underline{y}(Z)$ satisfy $|[G\Delta \underline{y}]^T [G\Delta \underline{y}] - 1| < \delta$, where δ is a predetermined constant. An iterative scheme is utilized to find such an optimizing coefficient, but is terminated after a specified number of iterations if an optimum value is not found. The matrix of bounds G is initialized for the iterative cycle. The modified G matrix depends upon the previously obtained value of $[\underline{B}^T \underline{B}]$; that is

$$\text{if } [\underline{B}^T \underline{B}]' < [\underline{B}^T \underline{B}], \text{ then } G = [m^{-1} \Delta \underline{y}]' \qquad \text{(reduction)}$$

or

$$\text{if } [\underline{B}^T \underline{B}]' > [\underline{B}^T \underline{B}], \text{ then } G = [m \Delta \underline{y}]' \qquad \text{(expansion)}$$

where

$\underline{B}^T \underline{B}$ is the dot-product of the vector of weighted residuals (the sum of squares)

G is the diagonal matrix of bounds

m is a prespecified constant

$\Delta \underline{y}$ is the correction vector

$'$ (prime) denotes the value (or vector) obtained from the previous iteration.

1. CONVERGENCE OF THE DIFFERENTIAL CORRECTION PROCESS

When a correction vector $\Delta \underline{y}(Z)$ is obtained in the iterative scheme described above, then examination is made of the convergence condition

$$\frac{\underline{B}^T \underline{B} - \| A \Delta \underline{y} - \underline{B} \|^2}{\underline{B}^T \underline{B}} \leq \epsilon$$

where $\underline{B} \neq \underline{0}$ and $\Delta \underline{y} \neq \underline{0}$ empirically (that is, linear techniques are used to solve a problem that is inherently nonlinear; therefore, neither residuals

nor differential corrections are identically zero), and ϵ is a prespecified relative convergence criterion. If this inequality is satisfied, the process has converged; if not, the iterative differential correction loop is continued.

Terms associated with convergence levels are "current rms" and "predicted rms," defined numerically as

$$\text{Current rms} = [(\underline{B}^T\underline{B})/n]^{1/2} \tag{39}$$

and

$$\text{Predicted rms} = (\|A\Delta\underline{\gamma} - \underline{B}\|^2/n)^{1/2} \tag{40}$$

2. SCHEMATIC REPRESENTATION OF THE DIFFERENTIAL CORRECTION PROCESS

The iterative differential correction loop depends on the differential corrector that performs the least squares process to obtain a set of corrections $\Delta\underline{\gamma}$ for the parameter $\underline{\gamma}$. Figure 7 is a schematic representation of the differential process with emphasis on the function of the differential corrector. The fundamental steps that identify the iterative differential correction loop can be summarized as follows:

Step 1. Establish initial values for the state vector $S_o(\underline{r}, \dot{\underline{r}})$ and parameter vector $\underline{\gamma}_o$

Step 2. Generate the vehicle's trajectory and associated partial derivatives from $S_o(\underline{r}, \dot{\underline{r}})$ and $\underline{\gamma}_o$

Step 3. Evaluate observation residuals Δ and partials $\partial 0^c/\partial\gamma$

Step 4. Perform residual editing and matrix accumulation

Step 5. Solve for the differential correction vector $\Delta\underline{\gamma}$ and test for convergence

Step 6. Apply correction; that is, reset $S_o(\underline{r}, \dot{\underline{r}})$ and $\underline{\gamma}_o = \underline{\gamma}_o + \Delta\underline{\gamma}$ and continue with Step 2.

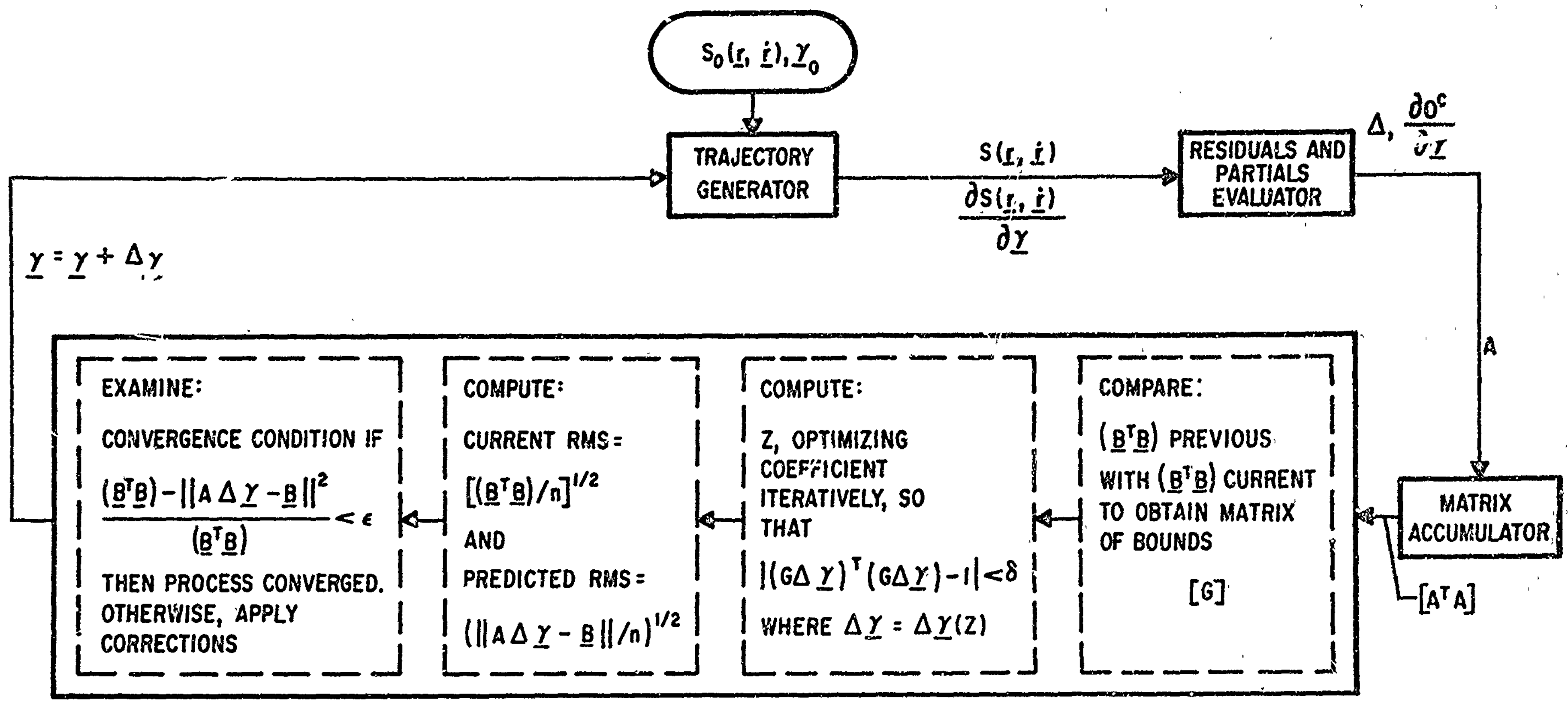

Figure 7. Schematic of the Differential Correction Process with Emphasis on the Differential Corrector

SECTION V

PROGRAM USAGE

The SPFP has features that permit the handling of data in several modes. Specific program control cards enable the user to edit otherwise inaccessible data, such as that stored on tape. Program deck setup depends upon the use of the control cards. The input or change of constants is accomplished by a generalized input routine.

A. DECK STRUCTURE AND FORMATS

The program deck setup depends upon the types of run desired and requires consideration of station location input, input of flags and constants, observation data flocks, and program control cards. Formats for each of these items are described in FORTRAN format notation.

1. STATION LOCATIONS

Up to 10 stations can be input to the program. The station cards are the first items read on any run and contain identifier and station coordinates (latitude, longitude, and height). The FORTRAN format is specified as (A3, 10X, 3E15.8). A station called TSb terminates the station card read (where b indicates a blank on the data card).

2. PRELIMINARY INPUT

Following the station location cards, the program reads initial inputs, which include all reference dates, flags, and constants. The program presets all flags and constants to perform a typical run with all observation-dependent constants in the basic unit system of feet, degrees, and feet per second. The epoch date corresponding to the earliest observation time is a recommended input. The format for this type of input consists of three fields, and as many as three sets of fields may be placed on one card. Each variable is identified

by a mnemonic of up to five characters. Mnemonics can also designate arrays, subsequent locations of which can be referenced by FORTRAN-type subscripts. The mnemonic and the values of the variable are left justified on the input load sheet. A one-character code is used to indicate the type of variable — b for real numbers (floating point) and I for integers (fixed point). The code E terminates data input reading. The FORTRAN format for the value is either E or F for real variables, and I for (left justified) integer variables. A detailed list of input mnemonics, their preset values, and descriptions is presented in Section V. 1. B.

3. OBSERVATION DATA INPUT

Observation data obtained from tracking a vehicle can be in either of two formats, corresponding to those used in the Aerospace Generalized Orbit Determination Program, TRACE, versions D (Ref. 1) and 66 (in preparation). Both formats contain station identification characters; the time the observation was measured (month, day, hour, minute, and second); observation type code; and three data fields containing R, A, and E measurements (type 1), or the first field containing an $\dot{R}$ measurement (type 7). The FORTRAN format for a TRACE-D data card is (A2, 2X, 4F3. 0, F7. 5, I1, 3F15. 8), and for a TRACE-66 data card (A3, 7X, 4F2. 0, F8. 5, 1X, I1, 2X, 3E15. 8). Each pass of data requires a station identification TRb to terminate the reading of observation data.

4. PROGRAM CONTROL CARDS

The SPFP control cards permit the user to control the program to fit his needs. There are four options that tell the program: to read a new set of flags or constants, to skip passes in the observation data input stream, to delete observations from a data pass, and to stop the program after processing the desired number of cases. It is fundamental to the use of these control cards that one card reside in memory and that a new card be read immediately after the resident control card is executed. The resident control card is continuously checked during the input processing of the data. When the conditions specified by the control card are met, the next card in the input stream

is expected to be another control card. The FORTRAN format for the program control card is (A10, 3I5). Table V lists the four available control cards in the SPFP.

Table V. Available Control Cards In the SPFP

Name	Case	Counters	Description
INPUT	N		Add new input to the N^{th} case
SKIP	N	K	Skip cases N through K (N < K)
DELETE	N	I J	Delete observations I through J in the N^{th} case
STOP	N		Stop after processing N^{th} case

The DELETE and SKIP cards are intended only for editing continuous input sources; i. e., tape.

Figures 8 and 9 indicate the deck setup for runs from data tapes and data cards, and are followed by sample listings for both.

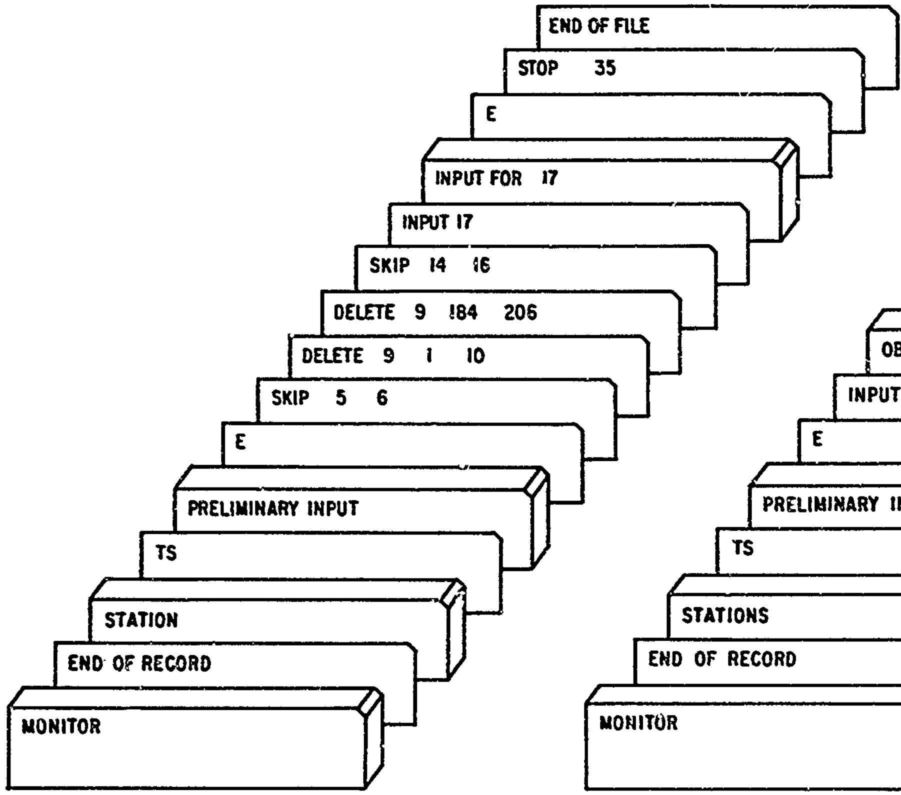

Figure 8. Sample Deck Setup Run from Tape

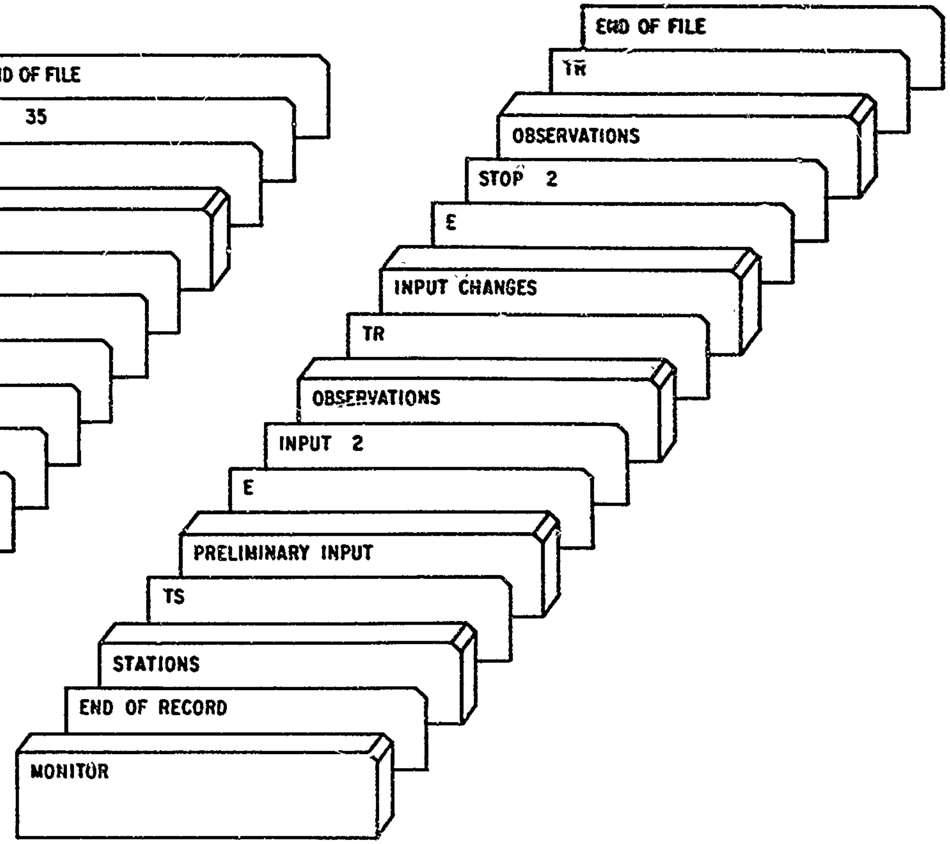

Figure 9. Sample Deck Setup Run from Cards

SAMPLE DECK SETUP RUN FROM CARDS

```
.
.    Monitor  Control  Cards
.    Program  Execution
     (End of Record Marker)
ST               36.71       311.65             176.5
TS
 RDAT   1967.                 2      8.                3     24.
IJT     5                    IIT     0                ISYSIN5
E
STØP             1
ST      8 24 16 43  0.0      1     3106728.0      2.198        2.185
ST      8 24 16 44  0.0      1     2034792.8      5.629       13.467
ST      8 24 16 45  0.0      1     1163245.6     31.678       53.629
ST      8 24 16 46  0.0      1     1186924.8    137.215       55.829
ST      8 24 16 47  0.0      1     2074629.3    174.329       14.367
ST      8 24 16 48  0.0      1     3279876.3    177.329        1.173
TR
     (End of File Marker)
```

SAMPLE DECK SETUP RUN FROM TAPE

```
.
.       Monitor   Control  Cards
.       Program   Execution
        (End of Record Marker)
STA               36.71           311.65          176.5
TS
 RDAT   1967.                     2     8.                        3     24.
IJT     2                        IIT    1                         ISYSIN5
E
SKIP              5    6
DELETE            9    1    10
SKIP             14   16
INPUT            17
 RDAT   1967                      3     26.                       IMAXIT8
E
STØP             20
        (End of File Marker)
```

B. INPUT DATA

All required input is nominally set to values in the basic unit system of feet, degrees, and feet per second. In the following list, each variable is described and the preset value designated in the value field.

PRESET CONSTANTS AND FLAGS

Code	Location	Value	Description
	RDAT	1964.	Epoch year
	2	1.	Month
	3	0.	Day
	REFE	.000312	Elevation refraction correction coefficient
	REFR	900.	Range refraction correction coefficient
	SCRNG	1.	Range scaling factor
	RMIN	0.	Minimum range
	RMAX	1.E30	Maximum range
	EMIN	0.	Minimum elevation
	TPASS	1440.	Maximum duration of pass (min)
I	SYSIN	5	Card reader input file
I	SYSØU	6	Printer output file
I	JT	2 (5)	Observation data input tape unit (card input)
I	IT	1 (0)	Data format TRACE-66 (-D)
I	NDIC	5	Initial condition determination selector
	EPS	.0001	Two-body orbit iteration convergence criterion
	SIGMA	100.	Range
	2	.1	Azimuth
	3	.1	Elevation
	4	0.	Range rate data weighting

Code	Location	Value		Description
	BNDS	50000.	x	Correction constraints
	2	50000.	y	
	3	50000.	z	
	4	500.	$\dot{x}$	
	5	500.	$\dot{y}$	
	6	500.	$\dot{z}$	
	7	0.	Drag	
	SCBN1	.5	Reduce	Bounds each iteration
	SCBN2	2.	Expand	
	EDITN	6.	n - σ rms residual editing factor	
	SCEDN	.8	Scale editing factor each iteration	
	EPSLN	.001	Relative convergence criterion	
	TR	50000.	Range	Initial residual acceptance limits
	2	5.	Azimuth	
	3	5.	Elevation	
	4	0.	Range rate	
I	MAXIT	5	Maximum number of iterations	
I	JPN	10	Number of observations to be generated ($\leq$ 10)	
I	IPN	1 (2)	Print (and punch) generated observations	
I	IW	1	0 print iteration summary only 1 print residuals 2 print the matrix of partial derivatives and residuals at each observation point	
	DF	20925738.	Distance conversion factor	
	VF	348762.3	Velocity conversion factor	
	F	.335233E-2	Earth oblateness coefficient (= 1/298.3)	
	GM	.55303935E-2	Gravitation constant	
	XNG	2.	n	28-term geopotential input
	XMG	0.	n	
	CNM	0.	C_{nm}	
	SNM	0.	S_{nm}	

Code	Location	Value	Description
I	NT	1	Number of geopotential terms
I	JNØRM	1	Normalization of geopotential coefficients
	H0	1.	Initial integration step size
	HMIN	0.015625	Minimum step size (1/64)
	HMAX	64.	Maximum step size
	ER	1.E-12	Integration truncation control

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13. ABSTRACT

The Single Pass Fit Program (SPFP) is designed for use on the CDC 6400 or 6600 machine as the principal computational tool for preliminary orbit determination and processing of single arcs of raw tracking data. By an iterative differential correction procedure, it solves for a set of trajectory parameters, namely, initial position and velocity (and drag coefficient if desired) of the vehicle, which minimize the differences between measured and computed observation measurements. A comprehensive description of the mathematical model implemented in the SPFP is presented with emphasis placed on key characteristics, such as reference systems and equations, data processing and self-initializing techniques, residual editing, and differential correction. In addition, a basic usage guide is given complete instructions for input data preparation and program operation.

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